

# A light sterile neutrino based on the seesaw mechanism

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## Abstract

We propose a simple model of the neutrino mass matrix which can explain the solar and atmospheric neutrino problems in a  $3(\nu_L + \nu_R)$  framework. Assuming that only two right-handed neutrinos are heavy and a Dirac mass matrix has a special texture, we construct a model with four light neutrinos. The favorable structure of flavor mixings and mass eigenvalues required by those neutrino deficits is realized as a result of the seesaw mechanism. Bi-maximal mixing structure might be obtainable in this scheme. Since it contains a light sterile neutrino, it has a chance to explain the LSND result successfully. We consider an embedding of this scenario for the neutrino mass matrix into the SU(5) grand unification scheme using the Froggatt-Nielsen mechanism based on  $U(1)_{F_1} \times U(1)_{F_2}$ . Both a small mixing angle MSW solution and a large mixing angle MSW solution are obtained for the solar neutrino problem depending on the charged lepton mass matrix.

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# 1. Introduction

Recently the existence of non-trivial lepton mixing has been strongly suggested through the atmospheric and solar neutrino observations whose results can be explained by assuming the neutrino oscillations [1, 2, 3]. The predicted flavor mixing is much bigger than the one of quark sector. The explanation of this feature is a challenging issue for the construction of a satisfactory grand unified theory (GUT) and a lot of works have been done [4, 5]. In most of them the smallness of the neutrino mass is explained by the celebrated seesaw mechanism [6] and the flavor mixing structure is considered to be controlled by the Froggatt-Nielsen mechanism [7]. There are many works in which the Abelian flavor symmetry is discussed [8]. On the other hand, there is another experimental suggestion on the neutrino oscillation by the Liquid Scintillator Neutrino Detector (LSND) [9]. If we impose the simultaneous explanation of the result together with the atmospheric and solar neutrino deficits, it has been well-known that three different values of the squared mass difference are necessary. Then four light neutrinos including a sterile neutrino ( $\nu_s$ ) are required [10]. Various models of the sterile neutrino can be found in refs. [10-14]. Following the recent Super-Kamiokande analysis of the solar neutrino, the explanation of the solar neutrino problem based on the  $\nu_e$ - $\nu_s$  oscillation seems to be disfavored [3]. It suggests that the (3+1)-neutrino spectrum might be a more favored scenario for the neutrino mass hierarchy than the (2+2)-scheme [15].

In this paper we consider a neutrino mass matrix in a  $3(\nu_L + \nu_R)$  framework by using the seesaw mechanism. However, being different from the ordinary seesaw models our model contains a light right-handed neutrino as a result of the special texture of a right-handed Majorana neutrino mass matrix. Although there are the similar works in this direction, in most of them it is necessary to introduce the Majorana masses for the left-handed neutrinos in order to obtain simultaneously the required values of the mass eigenvalues and the flavor mixing angles as it can be found, for example, in [13, 14]. It means that an introduction of a new triplet Higgs field might be necessary. In the present model we only need the Dirac neutrino masses and the right-handed Majorana neutrino masses if we assume a special but simple texture in both of them at tree level. The model seems to have less parameters as compared to the previous ones.

One of the interesting points of the model is that the large mixing angle MSW solution for the solar neutrino problem can be consistently accommodated in the same way as other

solutions [16, 17]. The LSND result might be also explained if we take an appropriate solution for the solar neutrino problem [17]. Moreover, it is interesting that this scenario for the neutrino mass matrix could also be embedded into the GUT scheme by introducing a suitable flavor symmetry. Such an example in the SU(5) model will be constructed by fixing the charge assignment of quarks and leptons for that symmetry.

The organization of this paper is as follows. In section 2 we define our model and discuss its various phenomenological features in the case that the charged lepton mass matrix is diagonal. In section 3 we consider the embedding of the scenario into the SU(5) GUT scheme. We discuss the realization of the required form of the mass matrix in the basis of the Froggatt-Nielsen mechanism. The flavor structure in the quark sector is also discussed here. Section 4 is devoted to the summary.

## 2. A model of neutrino mass matrix

We consider a model defined by the following neutrino mass terms which are different from the usual seesaw model in the  $3(\nu_L + \nu_R)$  framework:

$$-\mathcal{L}_{\text{mass}} = \sum_{\alpha} \sum_{p=2,3} m_{p\alpha} N_p \nu_{\alpha} + \sum_{p=2,3} m_{p1} N_p N_1 + \frac{1}{2} \sum_{p=2,3} M_p N_p N_p + \text{h.c.}, \quad (1)$$

where  $\nu_{\alpha}$  is an active neutrino ( $\alpha = e, \mu, \tau$ ) and  $N_P$  ( $P = 1 \sim 3$ ) is a charge conjugated state of the right-handed neutrino. We make the following assumption for the mass parameters in eq. (1):

$$m_{2e} = m_{2\mu} = m_{2\tau} \equiv \hat{\eta}, \quad m_{3e} \equiv \bar{\eta}_1, \quad m_{3\mu} = m_{3\tau} \equiv \bar{\eta}_2, \\ \hat{\eta} \sim \bar{\eta}_1 \sim \bar{\eta}_2 < m_{21} \sim m_{31} \ll M_2 \sim M_3, \quad (2)$$

where the mass parameters should be understood as their absolute values, although it is not expressed explicitly. A crucial difference from the usual seesaw model is that one of the right-handed neutrinos is assumed to be very light and also has very small mixings with other heavy right-handed neutrinos. We assume  $M_{23} = 0$  in the Majorana mass matrix of  $N_P$  here, for simplicity. Following arguments are not largely changed even if we introduce the non-zero  $M_{23}$ . Under this assumption we can integrate out heavy right-handed neutrinos  $N_p$  and get the following  $4 \times 4$  matrix as a result of the seesaw

mechanism<sup>1</sup>,

$$m_\nu = \begin{pmatrix} A & B & B & D \\ B & C & C & E \\ B & C & C & E \\ D & E & E & F \end{pmatrix}. \quad (3)$$

The matrix elements  $A \sim F$  are expressed by the model parameters in (2) as

$$\begin{aligned} A &= \frac{\hat{\eta}^2}{M_2} + \frac{\bar{\eta}_1^2}{M_3}, & B &= \frac{\hat{\eta}^2}{M_2} + \frac{\bar{\eta}_1 \bar{\eta}_2}{M_3}, & C &= \frac{\hat{\eta}^2}{M_2} + \frac{\bar{\eta}_2^2}{M_3}, \\ D &= \frac{\hat{\eta} m_{21}}{M_2} + \frac{\bar{\eta}_1 m_{31}}{M_3}, & E &= \frac{\hat{\eta} m_{21}}{M_2} + \frac{\bar{\eta}_2 m_{31}}{M_3}, & F &= \frac{m_{21}^2}{M_2} + \frac{m_{31}^2}{M_3}. \end{aligned} \quad (4)$$

If we define the diagonalization matrix  $U$  of the matrix (3) as  $m_\nu^{\text{diag}} = U^T m_\nu U$ , we find that  $U$  can be written as

$$U = \begin{pmatrix} \cos \theta & -\sin \theta & 0 & -\sin \theta \sin \delta + \cos \theta \sin \gamma \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} (\cos \theta \sin \delta + \sin \theta \sin \gamma) \\ \frac{1}{\sqrt{2}} \sin \theta & \frac{1}{\sqrt{2}} \cos \theta & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} (\cos \theta \sin \delta + \sin \theta \sin \gamma) \\ -\sin \gamma & -\sin \delta & 0 & 1 \end{pmatrix}, \quad (5)$$

where  $|\sin \gamma|, |\sin \delta| \ll 1$  is assumed and mixing angles are defined by

$$\tan 2\theta = \frac{2\sqrt{2}B}{A - 2C}, \quad \sin \gamma \simeq \frac{D \cos \theta + \sqrt{2}E \sin \theta}{F}, \quad \sin \delta \simeq \frac{-D \sin \theta + \sqrt{2}E \cos \theta}{F}. \quad (6)$$

The mass eigenvalues of  $m_\nu$  are expressed as

$$\begin{aligned} m_1 &\simeq A \cos^2 \theta + \sqrt{2}B \sin 2\theta + 2C \sin^2 \theta, \\ m_2 &\simeq A \sin^2 \theta - \sqrt{2}B \sin 2\theta + 2C \cos^2 \theta, \\ m_3 &= 0, \quad m_4 = F, \end{aligned} \quad (7)$$

where we neglect the contribution from the fourth low and column of  $m_\nu$  to  $m_{1,2}$  taking account of the fact such as  $A \gtrsim \frac{D^2}{F}$ ,  $B \gtrsim \frac{DE}{F}$  and  $C \gtrsim \frac{E^2}{F}$ . Here we should note that in

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<sup>1</sup>It should be noted that the number of light sterile neutrinos is restricted at most to one in the present scenario. We obtain a  $3 \times 3$  matrix if all of  $N_P$  are heavy. Even in such a case as far as we assume a proportional relation between  $(m_{1\alpha})$  and  $(m_{2\alpha})$  as vectors whose components are labeled by  $\alpha$ , the texture for the active neutrinos is the same as eq. (3). Then it can be applied to the explanation of the solar and atmospheric neutrino problems in the same way as the following discussion. It is essentially the same as the one discussed in ref. [18], although it is derived in the different context.

| $(\alpha, \beta)$ | $(i, j)$ | $-4U_{\alpha i}U_{\beta i}U_{\alpha j}U_{\beta j}(\equiv \mathcal{A})$ |     |
|-------------------|----------|--|-----|
| $(I, II)$         | (1,2)    | $\frac{1}{2}\sin^2 2\theta$  | (A) |
| $(I, III)$        | (1,2)    | $\frac{1}{2}\sin^2 2\theta$  | (B) |
| $(II, III)$       | (1,3)    | $\sin^2 \theta$  | (C) |
|                   | (2,3)    | $\cos^2 \theta$  | (D) |
|                   | (1,2)    | $-\frac{1}{4}\sin^2 2\theta$   | (E) |

Table 1. The contributions to each neutrino transition process  $\nu_\alpha \rightarrow \nu_\beta$  from each sector  $(i, j)$  of the mass eigenstates.

this model the violation of the proportional relation between  $(m_{2\alpha})$  and  $(m_{3\alpha})$  as vectors is crucial to restrict a number of zero mass eigenvalue into one and to control the mixing structure. There is a freedom in a choice of two elements of  $(m_{3\alpha})$  which are taken to be equal in (2). As far as we consider the case in which the charged lepton mass matrix is diagonal, it is not important. But when we consider the different situation, it might become crucial to the consideration of the oscillation phenomena. If the charged lepton mass matrix is diagonal, the above mixing matrix  $U$  is just the flavor mixing matrix  $V^{(\text{MNS})}$  which controls the neutrino oscillation. We assume it in the charged lepton sector and also no  $CP$  violation in the lepton sector. At this stage we cannot determine to which flavor each  $\nu_\alpha$  corresponds so that we will use the Roman numerals for the subscript  $\alpha$  for a while. Next we study the features of the oscillation phenomena in the model in order to fix the neutrino flavor.

The transition probability due to the neutrino oscillation  $\nu_\alpha \rightarrow \nu_\beta$  after the flight length  $L$  is well-known to be written by using the matrix elements of (5) as

$$\mathcal{P}_{\nu_\alpha \rightarrow \nu_\beta}(L) = \delta_{\alpha\beta} - 4 \sum_{i>j} U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j} \sin^2 \left( \frac{\Delta m_{ij}^2}{4E} L \right), \quad (8)$$

where  $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$  and the weak eigenstate  $\nu_\alpha$  is related to the mass eigenstate  $\tilde{\nu}_i$  by  $\nu_\alpha = U_{\alpha i} \tilde{\nu}_i$  in the basis that the charged lepton mass matrix is diagonal. In Table 1 we summarize the contribution to each neutrino transition mode  $(\alpha, \beta)$  from a sector  $(i, j)$  of the mass eigenstates. As a phenomenologically interesting case, we consider the situation that the mass eigenstates  $\tilde{\nu}_1$  and  $\tilde{\nu}_2$  are almost degenerate and the hierarchy  $(m_3 \ll m_1 \sim m_2) \ll m_4$  among the mass eigenvalues is satisfied. This corresponds to a

well-known reversed hierarchy scenario for the atmospheric and solar neutrino problems in the (3+1)-neutrino spectrum [19]. The absolute value of each mass eigenvalue is smaller than the ordinarily discussed scenario because of  $m_3 = 0$ . Then every neutrino cannot be a hot dark matter candidate. If we apply it to explain the atmospheric and solar neutrino data, the squared mass difference should be taken as [1, 2]

$$2 \times 10^{-3} \text{ eV}^2 \lesssim \Delta m_{13}^2 \simeq \Delta m_{23}^2 \lesssim 6 \times 10^{-3} \text{ eV}^2, \quad (9)$$

$$10^{-10} \text{ eV}^2 \lesssim \Delta m_{12}^2 \lesssim 1.5 \times 10^{-4} \text{ eV}^2. \quad (10)$$

A suitable value of  $\Delta m_{12}^2$  should be chosen within the above range depending on which solution is adopted for the solar neutrino problem.

By inspecting Table 1 we find that the simultaneous explanation of both deficits of the atmospheric neutrino and the solar neutrino is possible if we identify the weak eigenstates of neutrinos ( $e, \mu, \tau$ ) with ( $I, II, III$ ). Under this identification the  $3 \times 3$  submatrix of (5) is recognized as the correctly arranged MNS mixing matrix. If we note that  $m_3 = 0$  and  $\Delta m_{13}^2 \simeq \Delta m_{23}^2$  are satisfied, we find that the atmospheric neutrino is explained by  $\nu_\mu \rightarrow \nu_\tau$  obtained as the combination of (C) and (D) in Table 1. This explanation is independent of the value of  $\sin \theta$ . On the other hand, the solar neutrino is expected to be explained by  $\nu_e \rightarrow \nu_\mu$  (A) and also  $\nu_e \rightarrow \nu_\tau$  (B). In both processes the amplitude  $\mathcal{A} (\equiv -4 \sum U_{\alpha i} U_{\beta i} U_{\alpha j} U_{\beta j})$  is  $\frac{1}{2} \sin^2 2\theta$ . Thus if  $\sin^2 2\theta \sim 10^{-2}$ , the small mixing angle MSW solution (SMA) is realized [17]. In the case of  $\sin^2 2\theta \sim 1$ , it can give the large mixing angle MSW solution (LMA), the low mass MSW solution (LOW) and the vacuum oscillation solution (VO) [17] depending on the value of  $\Delta m_{12}^2$ . The CHOOZ experiment [20] constrains a component  $U_{e3}$  of the MNS mixing matrix [21]. It comes from the fact that the amplitude  $\mathcal{A}$  of the contribution to  $\nu_e \rightarrow \nu_x$  with the squared mass differences  $\Delta m_{13}^2$  or  $\Delta m_{23}^2$  always contains  $U_{e3}$ . The model is free from this constraint since  $U_{e3} = 0$  is satisfied independently of the value of  $\sin \theta$ .

In order to see the viability of the scenario in a more quantitative way it is useful to estimate numerically what kind of tuning of the primary parameters in (1) and (2) is required to realize the suitable value for the oscillation parameters. For the convenience we introduce the following parametrization for the three light states:

$$\frac{\hat{\eta}}{\sqrt{M_2}} \equiv \mu^{\frac{1}{2}}, \quad \frac{\bar{\eta}_1}{\sqrt{M_3}} \equiv \epsilon_1 \mu^{\frac{1}{2}}, \quad \frac{\bar{\eta}_2}{\sqrt{M_3}} \equiv \epsilon_2 \mu^{\frac{1}{2}}. \quad (11)$$

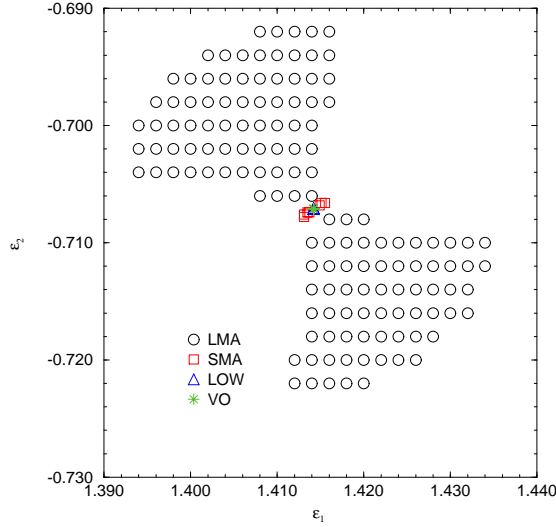


Fig. 1 The scatter plot of possible solutions for the atmospheric and solar neutrino problems in the  $(\epsilon_1, \epsilon_2)$  plane.

For simplicity, we assume  $M_2 = M_3$ . Then the overall mass scale is determined by  $\mu$  and the hierarchy among the mass eigenvalues is controlled by  $\epsilon_1$  and  $\epsilon_2$ . When  $\epsilon_1 = \sqrt{2}$  and  $\epsilon_2 = -\frac{1}{\sqrt{2}}$ , two mass eigenvalues  $m_1$  and  $m_2$  are degenerate. Using the fact, we can estimate a typical scale of  $\mu$  from the condition (9) as  $\mu \simeq 1.8 \times 10^{-2}$  eV. This value corresponds to  $M_2 \sim 5.6 \times 10^{10}$  GeV for  $\hat{\eta} \sim 1$  GeV. The deviation from these values of  $\epsilon_{1,2}$  determines the difference between  $m_1$  and  $m_2$  and also the value of  $\sin \theta$ . In Fig. 1 we give a scatter plot of the possible solutions for both the atmospheric and solar neutrino problems in the  $(\epsilon_1, \epsilon_2)$  plane. In this figure solutions for both sign of  $\sin 2\theta$  are contained. Since we consider the reversed hierarchy here, both possibilities are allowed. From the figure we find that the SMA, the LOW and the VO postulate the finer tuning of the parameters than the LMA to realize the required values of the squared mass difference and  $\sin^2 2\theta$ .

In the present model we have a light sterile neutrino. Therefore we may have a chance to explain the LSND result, if  $m_4 \sim \mathcal{O}(1)$  eV is satisfied. We can check whether all of mass eigenvalues and various mixing angles quoted in the above discussion can be consistent with the LSND explanation by using the required relations (4), (6) and (7). If we take  $\epsilon_1 \sim 1.41$  and  $\epsilon_2 \sim -0.71$  as a typical example in Fig. 1 and also assume  $M_2 = M_3$  and

$m_{21} = m_{31}$ , we obtain

$$\bar{\eta}_1 \sim 1.41\hat{\eta}, \quad \bar{\eta}_2 \sim -0.71\hat{\eta}, \quad \sin \gamma \sim 1.21 \frac{\hat{\eta}}{m_{31}}, \quad \sin \delta \sim 0.21 \frac{\hat{\eta}}{m_{31}}. \quad (12)$$

In order to see the feature related to the LSND we note that the relevant amplitude  $\mathcal{A}_{\text{LSND}}$  can be written by using the unitarity of  $V^{(\text{MNS})}$  and the relation  $|m_{1,2,3}| \ll |m_4|$  as

$$\mathcal{A}_{\text{LSND}} = 4(V_{e4}^{(\text{MNS})})^2(V_{\mu 4}^{(\text{MNS})})^2. \quad (13)$$

Then the amplitude can be written by using eq. (5) as

$$\mathcal{A}_{\text{LSND}} \simeq 2(\cos \theta \sin \gamma - \sin \theta \sin \delta)^2(\cos \theta \sin \delta + \sin \theta \sin \gamma)^2. \quad (14)$$

The LSND data require it to be in the range around  $1.2 \times 10^{-3}$  for  $\Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2$ . Here we should remind that  $|\sin \gamma|, |\sin \delta| \ll 1$  should be satisfied under our assumption (2). If we take the large mixing angle solutions for the solar neutrino problem, we obtain  $\mathcal{A}_{\text{LSND}} \sim \frac{1}{2}(\sin^2 \gamma - \sin^2 \delta)^2$ . Here we impose it to take the above mentioned value, we find  $m_{31} \sim 5.4\hat{\eta}$  and then  $\sin \gamma \sim 0.23$  and  $\sin \delta \sim 0.04$  by using eq. (12). Moreover,  $m_4$  can take a suitable value for the LSND result such as  $m_4 \sim \frac{2m_{31}^2}{M_2} \sim 1 \text{ eV}$ . On the other hand, if we adopt the SMA solution and then  $\cos \theta \sim 1$ , we have  $\mathcal{A}_{\text{LSND}} \sim 2 \sin^2 \gamma \sin^2 \delta$ . If we require it to take the suitable value, we find  $m_{31} \sim 3.2\hat{\eta}$  and  $m_4 \sim 0.4 \text{ eV}$  which is too small for the explanation of the LSND data. Taking account of these analyses, we find that the inclusion of the LSND result restricts our model to the large mixing angle solutions with respect to the solution for the solar neutrino problem. This feature of the model might be favorable if we take seriously the recent Super-Kamiokande analysis of the solar neutrino [3]. However, even in this case we should comment on an influence on the big-bang nucleosynthesis (BBN) due to the oscillation processes  $\nu_{\mu,\tau} \rightarrow \nu_s$  in the early universe. The required values of  $\sin \gamma$  and  $\sin \delta$  for the explanation of the LSND data induce these processes at a large rate. The BBN bound on  $\nu_{\mu,\tau} \rightarrow \nu_s$  given in ref. [22] cannot be satisfied unless we assume the presence of the large lepton number asymmetry at the BBN epoch [23].

In Table. 1 an only remaining contribution (E) to  $\nu_\mu \rightarrow \nu_\tau$  cannot imply any evidence in the short-baseline experiment even in the case of  $\sin^2 2\theta \simeq 1$  since  $\Delta m_{12}^2$  is too small. However, this mode may be relevant to the long-baseline experiment in the case of  $\Delta m_{12}^2 \sim 10^{-4} \text{ eV}^2$  which corresponds to the LMA solution of the solar neutrino deficit. We show



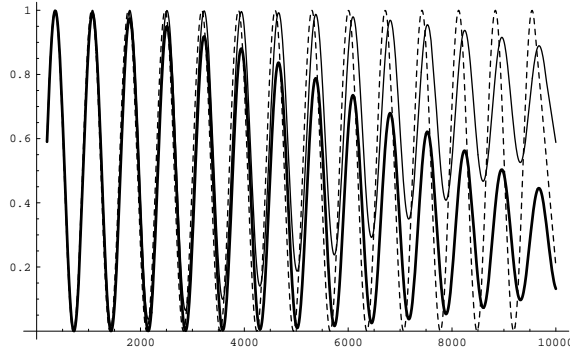


Fig. 2 The transition probability  $P(\nu_\mu \rightarrow \nu_{x(\neq\mu)})$  as a function of the flight length  $L$  km. We assume  $E = 1$  GeV,  $\Delta m_{13}^2 = 3.5 \times 10^{-3}$  eV<sup>2</sup> and  $\Delta m_{12}^2 = 10^{-4}$  eV<sup>2</sup>.

the effect of the mode (E) on the  $P(\nu_\mu \rightarrow \nu_x)$  in Fig. 2. The dashed line comes from the modes (C) and (D) which correspond to the ordinary two flavor oscillation  $\nu_\mu \rightarrow \nu_\tau$ . The thick solid line is the one which is obtained by taking account of the contribution of (E). In the thin solid line the contribution of (A) which corresponds to  $\nu_x = \nu_e$  is also taken into account. This shows that it may be possible to discriminate the model from others in the long-baseline experiment such as  $L \gtrsim 2000$  km. Moreover, the present model may be expected to have another experimental signature in the neutrinoless double  $\beta$ -decay [24]. Using eq. (5), the effective mass parameter which appears in a formula of the rate of neutrinoless double  $\beta$ -decay can be estimated as

$$|m_{ee}| \equiv \left| \sum_j |U_{ej}|^2 e^{i\phi_j} m_j \right| = (m_1 \cos^2 \theta + m_2 \sin^2 \theta) \sim m_1, \quad (15)$$

because of the fine degeneracy between  $m_1$  and  $m_2$ . Thus  $|m_{ee}|$  takes the value in the range of 0.04 - 0.08 eV which is independent of the value of  $\sin \theta$ , that is, the solution of the solar neutrino problem. This value seems to be within the reach in the near future experiments.

Before closing this section we order some short comments. In the above analysis we assume the reversed mass hierarchy in the (3+1) scheme. As another hierarchy among the mass eigenvalues we can consider the usual one, that is,  $(m_3 \lesssim m_2 \ll m_1) \ll m_4$  [15]. However, it is found that such a hierarchy cannot be consistent with the experimental data in the present scenario if we note that  $\nu_e$  cannot be identified with the state  $I$  to explain the solar neutrino deficit. We should also note that the above result crucially depends on the assumption on the charged lepton sector, for which we assumed that its

mass matrix was diagonal. As far as the flavor mixing in the charged lepton sector is small, the above result seems to be applicable. However, if it is large, the result can be largely changed. When we consider the GUT, such situations happen. In the next section we study this issue.

### 3. Embedding into SU(5)

We consider an embedding of our neutrino model into the supersymmetric SU(5) GUT. In that case the charged lepton mass matrix can be related to the neutrino mass matrix through the group theoretical constraint. Thus we cannot assume the small flavor mixing in the quark sector independently of the lepton sector. The result obtained in the previous section may be modified if we embed our scenario into the GUT scheme. In order to control the flavor mixing structure we adopt the Froggatt-Nielsen mechanism and introduce Abelian flavor symmetries  $U(1)_{F_1} \times U(1)_{F_2}$ . The symmetries are assumed to be broken by small parameters  $\lambda$  and  $\epsilon$  so that Yukawa couplings inducing the fermion masses are suppressed by both powers of  $\lambda$  and  $\epsilon$ . As a result the fermion mass hierarchy is produced. We take a model discussed in [5] as such a typical example and modify it to embed our scenario for the neutrino mass into it.

In the SU(5) GUT quarks and leptons are embedded into the representations of SU(5) as follows,

$$\mathbf{10} \ni (q, u^c, e^c), \quad \mathbf{5}^* \ni (d^c, \ell), \quad \mathbf{1} \ni \nu^c. \quad (16)$$

We assign the charge of  $U(1)_{F_1} \times U(1)_{F_2}$  to each representation in the following way [5]:

$$\begin{aligned} \mathbf{10} & : & (3, 2, 0), & (0, 0, 0), \\ \mathbf{5}^* & : & (c, 0, 0), & (0, 0, 0), \\ \mathbf{1} & : & (0, 0, 0), & (\alpha, \beta, \beta), \end{aligned} \quad (17)$$

where the numbers in the parentheses represent the charges given to each generation and  $c, \alpha$  and  $\beta$  are non-negative integers. The ordinary doublet Higgs fields  $H_1$  and  $H_2$  in the minimal supersymmetric standard model are assumed to have no charge of  $U(1)_{F_1} \times U(1)_{F_2}$ . In addition to these fields we introduce SU(5) singlet fields  $S_1$  and  $S_2$  which have the charges  $(-1, 0)$  and  $(0, -1)$  of the flavor symmetries, respectively<sup>2</sup>. The

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<sup>2</sup>We may need to introduce some fields to cancel the chiral anomaly of the flavor symmetry if it is a non-anomalous gauge symmetry. However, we do not go further into this issue in the present paper.

symmetries control the flavor mixing structure by regulating the number of fields  $S_1$  and  $S_2$  contained in each non-renormalizable term. If the singlet fields  $S_1$  and  $S_2$  get the vacuum expectation values  $\langle S_1 \rangle$  and  $\langle S_2 \rangle$ , the above mentioned suppression factors for the Yukawa couplings can be realized as the power of  $\lambda = \frac{\langle S_1 \rangle}{M_{\text{pl}}}$  and  $\epsilon = \frac{\langle S_2 \rangle}{M_{\text{pl}}}$ . Here  $M_{\text{pl}}$  is the Planck scale.

Using the Abelian flavor charges introduced above, we can obtain the quark and lepton mass matrices in the following form:

$$\begin{aligned}
M_u &\sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H_2 \rangle, & M_d &\sim \begin{pmatrix} \lambda^{3+c} & \lambda^{2+c} & \lambda^c \\ \lambda^3 & \lambda^2 & 1 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \langle H_1 \rangle, \\
M_\nu &\sim \begin{pmatrix} \lambda^c \epsilon^\alpha & \epsilon^\alpha & \epsilon^\alpha \\ \lambda^c \epsilon^\beta & \epsilon^\beta & \epsilon^\beta \\ \lambda^c \epsilon^\beta & \epsilon^\beta & \epsilon^\beta \end{pmatrix} \langle H_2 \rangle, & M_e &\sim \begin{pmatrix} \lambda^{3+c} & \lambda^3 & \lambda^3 \\ \lambda^{2+c} & \lambda^2 & \lambda^2 \\ \lambda^c & 1 & 1 \end{pmatrix} \langle H_1 \rangle, \\
M_R &\sim \begin{pmatrix} \epsilon^{2\alpha} & \epsilon^{\alpha+\beta} & \epsilon^{\alpha+\beta} \\ \epsilon^{\alpha+\beta} & \epsilon^{2\beta} & \epsilon^{2\beta} \\ \epsilon^{\alpha+\beta} & \epsilon^{2\beta} & \epsilon^{2\beta} \end{pmatrix} M,
\end{aligned} \tag{18}$$

where  $M$  is the mass scale relevant to the origin of the right-handed Majorana neutrino mass. Dirac mass matrices are written in the basis of  $\bar{\psi}_R m_D \psi_L$ . We do not consider the CP phases here. In the mass matrices (18) we abbreviate the order one coupling constants by using the similarity symbol. We should note that  $M_\nu$  and  $M_R$  in (18) can have a similar texture to the one defined by eqs. (1) and (2) up to the implicit coefficients of order one as far as  $\alpha \gg \beta$  is satisfied<sup>3</sup>. At least in the case of  $c = 0$  and 1 which is assumed in the following discussion, we can make  $M_\nu$  satisfy the condition (2) by tuning the order one coefficients. Thus we can have the similar mass matrix to eq. (3) as a result of the seesaw mechanism, although there is a non-zero element  $M_{23}$  in  $M_R$  differently from the one defined by eq. (1). Its diagonalization matrix  $U$  can be considered to have the similar form as eq. (5). Their difference comes only from the definition of the matrix elements  $A \sim F$  as we will see it below.

In the quark sector the mass eigenvalues and the CKM matrix elements can be found

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<sup>3</sup> In this context the order one coefficients assumed here may be allowed to be considered in the range  $(\sqrt{\epsilon}, \frac{1}{\sqrt{\epsilon}})$  if  $\epsilon < \lambda$  for  $M_\nu$ .

after some inspection as

$$m_u : m_c : m_t = \lambda^6 : \lambda^4 : 1, \quad m_d : m_s : m_b = \lambda^{3+c} : \lambda^2 : 1, \quad (19)$$

$$V_{us} \sim \lambda, \quad V_{ub} \sim \lambda^3, \quad V_{cb} \sim \lambda^2. \quad (20)$$

On the charged lepton sector we can know the mass eigenvalues by noting the SU(5) relation such as  $M_e^T = M_d$ . The ratio of mass eigenvalues is the same as the one of the down quark sector and then

$$m_e : m_\mu : m_\tau = \lambda^{3+c} : \lambda^2 : 1. \quad (21)$$

The result has some different features from the ones presented in ref. [5] in the down quark and charged lepton sectors. It comes from the charge assignment for  $\mathbf{5}^*$  which is needed to realize the Dirac neutrino masses defined by eqs. (1) and (2). If we assume  $\lambda \sim 0.22$ , these results seem to describe the experimental data in a qualitatively favorable way, except for  $m_e$  and  $m_u$  which are predicted to be too large, in particular, in the case of  $c = 0$ . This is the common fault known in the scheme based on the Abelian flavor symmetry and its similar charge assignment to the one given in (17). We cannot overcome it without something new.

We define the diagonalization matrix  $\tilde{U}$  of the charged lepton mass matrix in a basis that  $\tilde{U}^\dagger M_e^\dagger M_e \tilde{U}$  is diagonal. Then  $\tilde{U}$  can be approximately written as

$$c = 0 : \quad \tilde{U} = \begin{pmatrix} \frac{1}{\sqrt{2}} \cos \xi - \frac{1}{\sqrt{6}} \sin \xi & \frac{1}{\sqrt{2}} \sin \xi + \frac{1}{\sqrt{6}} \cos \xi & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{2}} \cos \xi - \frac{1}{\sqrt{6}} \sin \xi & -\frac{1}{\sqrt{2}} \sin \xi + \frac{1}{\sqrt{6}} \cos \xi & \frac{1}{\sqrt{3}} & 0 \\ \frac{2}{\sqrt{6}} \sin \xi & -\frac{2}{\sqrt{6}} \cos \xi & \frac{1}{\sqrt{3}} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad (22)$$

$$c = 1 : \quad \tilde{U} = \begin{pmatrix} \cos \xi & 0 & \sin \xi & 0 \\ -\frac{1}{\sqrt{2}} \sin \xi & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \xi & 0 \\ -\frac{1}{\sqrt{2}} \sin \xi & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \cos \xi & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \quad (23)$$

The hierarchical structure (21) of the mass eigenvalues requires a mixing angle  $\xi$  to be  $\sin \xi \sim O(\lambda)$ . In the neutrino sector we need to determine the finer structure of the Dirac neutrino mass matrix to be suitable for the charged lepton mass matrix given in (18) from a viewpoint of the explanation of various data for the neutrino oscillations.

For that purpose we should remind that there is a freedom in the choice of two elements of Dirac neutrino masses ( $m_{3\alpha}$ ), which are taken to be equal by tuning of the order one coefficients. After some investigation we find that it seems to be favorable to take  $m_{3e} = m_{3\mu} = \bar{\eta}_1$ ,  $m_{3\tau} = \bar{\eta}_2$  instead of the one given in eq. (2)<sup>4</sup>. Under this assumption the mass matrix of the light neutrinos can be written as

$$m_\nu = \begin{pmatrix} A & A & B & D \\ A & A & B & D \\ B & B & C & E \\ D & D & E & F \end{pmatrix}, \quad (24)$$

and the matrix elements  $A \sim F$  are defined by

$$\begin{aligned} A &= \frac{\hat{\eta}^2}{\bar{M}_3} + \frac{\bar{\eta}_1^2}{\bar{M}_2} - 2\frac{\hat{\eta}\bar{\eta}_1}{\bar{M}_{23}}, & B &= \frac{\hat{\eta}^2}{\bar{M}_3} + \frac{\bar{\eta}_1\bar{\eta}_2}{\bar{M}_2} - \frac{\hat{\eta}(\bar{\eta}_1 + \bar{\eta}_2)}{\bar{M}_{23}}, \\ C &= \frac{\hat{\eta}^2}{\bar{M}_3} + \frac{\bar{\eta}_2^2}{\bar{M}_2} - 2\frac{\hat{\eta}\bar{\eta}_2}{\bar{M}_{23}}, & D &= \frac{\hat{\eta}m_{21}}{\bar{M}_3} + \frac{\bar{\eta}_1m_{31}}{\bar{M}_2} - \frac{\hat{\eta}m_{31} + \bar{\eta}_1m_{21}}{\bar{M}_{23}}, \\ E &= \frac{\hat{\eta}m_{21}}{\bar{M}_3} + \frac{\bar{\eta}_2m_{31}}{\bar{M}_2} - \frac{\hat{\eta}m_{31} + \bar{\eta}_2m_{21}}{\bar{M}_{23}}, & F &= \frac{m_{21}^2}{\bar{M}_3} + \frac{m_{31}^2}{\bar{M}_2} - 2\frac{m_{21}m_{31}}{\bar{M}_{23}}, \end{aligned} \quad (25)$$

where  $\bar{M}_a^{-1} = M_a/(M_2M_3 - M_{23}^2)$  and we partially use the notation in eq. (2). The mass eigenvalues of (24) are

$$\begin{aligned} m_1 &\simeq 2A \cos^2 \theta + \sqrt{2}B \sin 2\theta + C \sin^2 \theta, \\ m_3 &\simeq 2A \sin^2 \theta - \sqrt{2}B \sin 2\theta + C \cos^2 \theta, \\ m_2 &= 0, \quad m_4 = F, \end{aligned} \quad (26)$$

where we again neglect the additional contributions to  $m_{1,3}$  because of the same reason as the one in the previous section. The diagonalization matrix  $U$  is rearranged from eq. (5) because of the change in the choice of  $\bar{\eta}_1$  and  $\bar{\eta}_2$ . Using the modified  $U$  and eqs. (22) and (23), the MNS matrix of the lepton mixing defined by  $V^{(\text{MNS})} = \tilde{U}^T U$  is calculated for both values of  $c$  as

$$V_{c=0}^{(\text{MNS})} \simeq \begin{pmatrix} -\frac{f_-^{(1)}}{\sqrt{3}} \sin \xi & -\cos \xi & \frac{f_+^{(2)}}{\sqrt{3}} \sin \xi & a_1 \\ \frac{f_-^{(1)}}{\sqrt{3}} \cos \xi & -\sin \xi & -\frac{f_+^{(2)}}{\sqrt{3}} \cos \xi & a_2 \\ \frac{f_+^{(2)}}{\sqrt{3}} & 0 & \frac{f_-^{(1)}}{\sqrt{3}} & a_3 \\ -\sin \gamma & 0 & -\sin \delta & 1 \end{pmatrix}, \quad (27)$$

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<sup>4</sup> It means that  $\hat{\eta} \sim \bar{\eta}_1 \sim \lambda^c \epsilon^\beta \sim \epsilon^\beta$  and  $\bar{\eta}_2 \sim \epsilon^\beta$ . The difference among them comes from the order one coefficients.

$$V_{c=1}^{(\text{MNS})} \simeq \begin{pmatrix} \frac{\cos \theta}{\sqrt{2}} \cos \xi - \frac{f_+^{(1)}}{2} \sin \xi & -\frac{1}{\sqrt{2}} \cos \xi & -\frac{\sin \theta}{\sqrt{2}} \cos \xi - \frac{f_-^{(2)}}{2} \sin \xi & a_1 \\ \frac{f_-^{(1)}}{2} & \frac{1}{2} & -\frac{f_+^{(2)}}{2} & a_2 \\ \frac{\cos \theta}{\sqrt{2}} \sin \xi + \frac{f_+^{(1)}}{2} \cos \xi & \frac{1}{2} \cos \xi & -\frac{\sin \theta}{\sqrt{2}} \sin \xi + \frac{f_-^{(2)}}{2} \cos \xi & a_3 \\ -\sin \gamma & 0 & -\sin \delta & 1 \end{pmatrix}, \quad (28)$$

where we use definitions

$$a_i = v_{i1} \sin \gamma + v_{i3} \sin \delta, \quad f_{\pm}^{(1)} = \cos \theta \pm \sqrt{2} \sin \theta, \quad f_{\pm}^{(2)} = \sqrt{2} \cos \theta \pm \sin \theta, \quad (29)$$

and  $v_{ij}$  represents the  $ij$ -element of the corresponding  $V^{(\text{MNS})}$ . To derive these expressions we use  $|\sin \gamma|, |\sin \delta| \ll 1$  and neglect higher order terms of them. The mixing angles  $\theta, \gamma$  and  $\delta$  in this case are defined as

$$\tan 2\theta = \frac{2\sqrt{2}B}{2A - C}, \quad \sin \gamma \simeq \frac{\sqrt{2}D \cos \theta + E \sin \theta}{F}, \quad \sin \delta \simeq \frac{-\sqrt{2}D \sin \theta + E \cos \theta}{F}. \quad (30)$$

Now we study the oscillation phenomena in both cases in more detail. First we consider the case of  $c = 0$ . Taking account that  $m_2 = 0$  and  $V_{\tau 2}^{(\text{MNS})} = 0$ , the reversed hierarchy scenario cannot be adopted from a viewpoint of the atmospheric neutrino problem. We must assume the normal hierarchy ( $|m_2| \lesssim |m_1| \ll |m_3| \ll |m_4|$ ) in the (3+1)-scheme in order to realize  $\Delta m_{12}^2 \simeq \Delta m_{\text{solar}}^2$  and  $\Delta m_{23}^2 \simeq \Delta m_{13}^2 \simeq \Delta m_{\text{atm}}^2$ . Using eq. (27), we find that the amplitude for  $\nu_\mu \rightarrow \nu_\tau$  is

$$\mathcal{A} = -4V_{\mu 1}^{(\text{MNS})}V_{\tau 1}^{(\text{MNS})}V_{\mu 3}^{(\text{MNS})}V_{\tau 3}^{(\text{MNS})} = \frac{4}{9}(\sqrt{2} - \frac{1}{2}\tan 2\theta)^2 \cos^2 2\theta \cos^2 \xi. \quad (31)$$

After some investigation we find that it suggests that  $\tan 2\theta \leq 0$  and  $\cos^2 \theta \sim 1$  should be satisfied for the explanation of the atmospheric neutrino problem. We should also remind the fact that  $\sin \xi \sim O(\lambda)$ . Although the large value of  $|\sin \theta|$  such as 0.95 can satisfy the bound from the atmospheric neutrino, it seems to be disfavored by the solar neutrino data. Thus the atmospheric and solar neutrino problems can be explained by  $\nu_\mu \rightarrow \nu_\tau$  corresponding to  $\Delta m_{13}^2$  and  $\nu_e \rightarrow \nu_\mu$  corresponding to  $\Delta m_{12}^2$ , respectively. The situation for the solar neutrino problem is different from the case in the previous section. Only the SMA solution is allowed in the present case since  $V_{\tau 2}^{(\text{MNS})} = 0$  and  $m_2 = 0$  make the contribution of  $\nu_e \rightarrow \nu_\tau$  to solar neutrino deficit zero. It originally comes from the non-diagonal structure of the charged lepton mixing matrix (22). Since the present neutrino mass matrix induces the large mixing between  $\nu_e$  and  $\nu_\mu$  by itself, we need the small

mixing in a corresponding place of the charge lepton sector to realize the large mixing solution for the solar neutrino. However, it is not satisfied there in this case.

In order to see the viability of the model quantitatively we need to check numerically the consistent realization of both data of the atmospheric and solar neutrino observations. In this study, for simplicity, we assume  $M_{23} = 0$  and  $M_2 = M_3$  here, although only the symmetries  $U(1)_{F_1} \times U(1)_{F_2}$  cannot verify the former one. Then we can use the parametrization (11). In Fig. 3 we give the scatter plot of solutions for both the atmospheric and solar neutrino problems in the  $(\epsilon_1, \epsilon_2)$  plane assuming  $-0.42 \leq \tan 2\theta \leq 0$ . Using the figure, we can find the typical values of the primary parameters in the model by using eqs. (25), (26) and (30). As an example, if we take  $\epsilon_1 \sim 3.0$  and  $\epsilon_2 \sim -0.45$  from Fig. 3, we can obtain

$$\bar{\eta}_1 \sim 3.0\hat{\eta}, \quad \bar{\eta}_2 \sim -0.45\hat{\eta}, \quad \tan 2\theta \sim -0.05, \quad \sin \gamma \sim 2.8 \frac{\hat{\eta}}{m_{31}}, \quad \sin \delta \sim 0.35 \frac{\hat{\eta}}{m_{31}}. \quad (32)$$

If we assume  $\cos \xi = 0.98$  and  $\sin \xi = 0.2$ , in this case the MNS matrix becomes

$$V_{c=0}^{(\text{MNS})} = \begin{pmatrix} -0.12 & -0.98 & 0.16 & -0.12 \sin \gamma + 0.16 \sin \delta \\ 0.59 & -0.20 & -0.78 & 0.59 \sin \gamma - 0.78 \sin \delta \\ 0.80 & 0 & 0.60 & 0.80 \sin \gamma + 0.60 \sin \delta \\ -\sin \gamma & 0 & -\sin \delta & 1 \end{pmatrix}. \quad (33)$$

The CHOOZ constraint on  $V_{e3}^{(\text{MNS})}$  is satisfied. The LSND result may have a chance to be again explained because of the existence of one light sterile neutrino. In order to see it we study the relevant amplitude  $\mathcal{A}_{\text{LSND}}$  which is estimated by eq. (13). If we require  $\mathcal{A}_{\text{LSND}} \sim 1.2 \times 10^{-3}$ , we obtain  $m_{31} \sim 4.7\hat{\eta}$  and then  $m_4 \sim 0.1$  eV where we take  $\mu \sim 2.6 \times 10^{-3}$  eV. This is too small to explain the LSND data. Although the larger value of  $\mathcal{A}_{\text{LSND}}$  induces the smaller value of  $m_4$ , we cannot find a favorable result for the LSND within the present freedom. In the present case  $\mu$  and  $V_{e1}^{(\text{MNS})}$  tends to take small values by the requirement of the atmospheric and solar neutrinos. As a result of this general feature,  $m_4$  takes a small value compared to the required value by the LSND. The effective mass parameter  $|m_{ee}|$  for the neutrinoless double  $\beta$ -decay can be estimated as  $|m_{ee}| \sim U_{e4}^2 m_4 \sim 0.16\mu$ . It is too small as compared with the value expected to be reached by the near future experiment.

It is also useful to note that the above values of the primary parameters of the model are realized through the suitable charge assignment of  $\alpha$  and  $\beta$ . In order to show such an

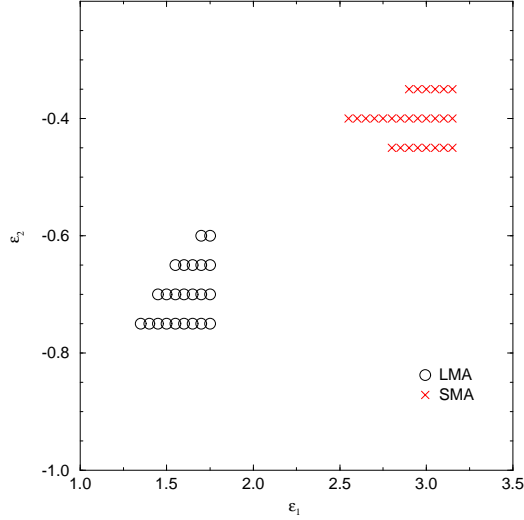


Fig. 3 The scatter plot of possible solutions for both of the solar and atmospheric neutrino problems in the  $(\epsilon_1, \epsilon_2)$  plane. Requiring  $-0.42 \leq \tan 2\theta \leq 0$ , the scale parameter  $\mu$  is taken as  $7.7 \times 10^{-3}$  eV for the LMA and  $2.6 \times 10^{-3}$  eV for the SMA.

example we take  $\langle H_2 \rangle \sim 100$  GeV,  $M \sim 4 \times 10^{15}$  GeV and  $\epsilon \sim 10^{-2}$ . Then if we assign  $\alpha = 6$  and  $\beta = 0^5$ , we can check that all required quantities except for  $\mathcal{A}_{\text{LSND}}$  are realized in the suitable range discussed numerically above up to the order one factors. In this case we have  $m_4 \sim 1$  eV.

Next we treat the case of  $c = 1$ . Also in this case the reversed hierarchy cannot induce the sufficiently large amplitude  $\mathcal{A}$  for  $\nu_\mu \rightarrow \nu_\tau$  using the modes with  $\Delta m_{12}^2$  and  $\Delta m_{23}^2$ . We need adopt the normal mass hierarchy also in the present case. The relevant amplitude for  $\nu_\mu \rightarrow \nu_\tau$  is estimated as

$$\mathcal{A} = \sum_{i=1,2} -4V_{\mu i}^{(\text{MNS})}V_{\tau i}^{(\text{MNS})}V_{\mu 3}^{(\text{MNS})}V_{\tau 3}^{(\text{MNS})} = \left( \cos^2 \theta - \frac{\sin^2 \theta}{2} \right) \left( \frac{\cos^2 \theta}{2} - \sin^2 \theta + \frac{1}{2} \right), \quad (34)$$

where we take account of  $\sin \xi \ll 1$  in the estimation. If  $\cos^2 \theta \sim 1$  is satisfied, the above amplitude can be suitable to the atmospheric neutrino problem. The contribution to the solar neutrino deficits comes from  $\nu_e \rightarrow \nu_\mu$  and  $\nu_e \rightarrow \nu_\tau$  with  $\Delta m_{12}^2$ . Their combined amplitude is almost equal to one and the large mixing angle solution is realized. The value of  $\cos^2 \theta$  is fixed to generate the large mixing for the explanation of the atmospheric

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<sup>5</sup>The more general condition is  $\epsilon^\alpha = 10^{-12}$ . Since  $\epsilon$  can be taken as a very small value, it can be consistent even if the coefficients which are considered to be of order one are rather large.



neutrino and it also results in the large mixing between  $\nu_e$  and  $\nu_\mu$ . On the other hand, in the charged lepton sector  $c = 1$  makes the mixing between  $e$  and  $\mu$  small so that we can have a large mixing angle solution for the solar neutrino. Also in this case we give the scatter plot of the LMA solutions in Fig. 3 by assuming  $M_{23} = 0$  and  $M_2 = M_3$  and using the parametrization (11). The LOW and VO solutions seem to be difficult to be realized since the required  $\Delta m_{12}^2$  should be much smaller than the LMA. Using Fig. 3 we can again find the typical values of the primary parameters in the model. As an example, if we take  $\epsilon_1 \sim 1.75$  and  $\epsilon_2 \sim -0.75$ , we can obtain

$$\bar{\eta}_1 \sim 1.75\hat{\eta}, \quad \bar{\eta}_2 \sim -0.75\hat{\eta}, \quad \tan 2\theta \sim -0.13, \quad \sin \gamma \sim 1.9 \frac{\hat{\eta}}{m_{31}}, \quad \sin \delta \sim 0.26 \frac{\hat{\eta}}{m_{31}}. \quad (35)$$

These fix the MNS matrix in the present case as

$$V_{c=1}^{(\text{MNS})} = \begin{pmatrix} 0.62 & -0.79 & -0.10 & 0.62 \sin \gamma - 0.10 \sin \delta \\ 0.55 & 0.50 & -0.67 & 0.55 \sin \gamma - 0.67 \sin \delta \\ 0.58 & 0.35 & 0.73 & 0.58 \sin \gamma + 0.73 \sin \delta \\ -\sin \gamma & 0 & -\sin \delta & 1 \end{pmatrix}. \quad (36)$$

We can see that the CHOOZ constraint on  $V_{e3}^{(\text{MNS})}$  is satisfied in (36). In order to see the possibility to explain the LSND result we impose  $\mathcal{A}_{\text{LSND}} \sim 1.2 \times 10^{-3}$ . Then by using eq. (13) and  $\mu \sim 7.7 \times 10^{-3}$  eV, we obtain  $m_{31} \sim 7.8\hat{\eta}$  and then  $m_4 \sim 0.93$  eV which seems to be in the suitable region for the LSND. We also have  $\sin \gamma \sim 0.25$  and  $\sin \delta \sim 0.03$  which are consistent with our assumption for  $\sin \gamma$  and  $\sin \delta$ . These analyses show that this case can explain all neutrino oscillation data including the LSND. It is also interesting that the effective mass  $|m_{ee}|$  for the neutrinoless double  $\beta$ -decay can have rather large value because of the  $m_4$  contribution. In fact, using the above numerical values it can be estimated as  $|m_{ee}| \sim |U_{e4}|^2 m_4 \sim 0.02$  eV, which may be a promising value from the experimental viewpoint. The value of the charges  $\alpha$  and  $\beta$  adopted for the  $c = 0$  case is also applicable to the present case to realize the model parameters in a favored region up to the order one coefficients as far as we take the same values of  $\langle H_2 \rangle \sim 100$  GeV,  $M \sim 10^{15}$  GeV and  $\epsilon \sim 10^{-2}$ . However, the present case needs more complicated structure of the order one coefficients as compared with the  $c = 0$  case.

Finally, we should note that in our scheme the required equality among  $(m_{2\alpha})$  and also among  $(m_{3\alpha})$  in eq. (2) might not be necessary to be satisfied exactly. The allowed

deviation should be quantitatively investigated since it is related to the estimation of the magnitude of order one coefficients.

## 4. Summary

We have proposed the scenario for the neutrino mass and mixing based on the seesaw mechanism in the  $3(\nu_L + \nu_R)$  framework. By assuming the special texture for the right-handed Majorana neutrino mass matrix and the Dirac mass matrix we could obtain a model with four light neutrino states including a sterile neutrino. One active neutrino is massless and others can have the masses which are suitable for the explanation of the atmospheric and solar neutrino deficits, and also the LSND result. We studied two different cases specified by the diagonal charged lepton mass matrix and the non-diagonal one which is obtained by embedding of our neutrino mass matrix into the SU(5) GUT scheme.

In the former case the so-called reversed mass hierarchy scenario has been adopted. Every known solution for the solar neutrino problem could be realized by tuning the Dirac mass matrix of neutrinos. It is interesting that the large mixing angle MSW solution can be most easily realized as compared to other solutions. Moreover, if we impose the explanation of the LSND on it, the solution for the solar neutrino problem is restricted to the ones with the large mixing angle. The difference from the two flavor oscillation could be expected to be observed in the  $\nu_\mu \rightarrow \nu_\tau$  using the long-baseline experiment with the flight length more than 2000km. The neutrinoless double  $\beta$ -decay might also be accessible if the experimental bound is improved to the level of  $|m_{ee}| \sim 0.04 - 0.08$  eV.

In the latter case the Froggatt-Nielsen mechanism has been applied to control the flavor mixing. We introduce the Abelian flavor symmetries  $U(1)_{F_1} \times U(1)_{F_2}$  whose factor groups are assumed to have the different breaking scale. The non-trivial charge assignment of  $U(1)_{F_1}$  is used only for the **10** and **5\*** fields of SU(5) and the right-handed neutrino **1** is assumed to have only the charge of  $U(1)_{F_2}$ . Under this setting we studied the features of the mass and the mixing in both quark and lepton sectors for the two types of the charge assignment. We found that it could generate the mass eigenvalues and the flavor mixings for the quark sector in a qualitatively satisfactory way. If we give the suitable flavor charge to the right-handed neutrinos, our neutrino scenario can be also embedded into the SU(5) scheme consistently. Although the reversed mass hierarchy is disfavored in

both charge assignments, the ordinary mass hierarchy presents a consistent explanation of all data of the known neutrino oscillation observations. For the solar neutrino problem only the SMA solution or the LMA solution is allowed in each case. However, if we impose the explanation of the LSND result, only the LMA seems to be favored.

In this paper we assumed that the tuning of order one coefficients could always be allowed. Although in our scenario the mild tuning of order one coefficients is very crucial, we cannot say anything on its origin at the present stage.

## **Acknowledgement**

This work is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture (No.11640267).

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